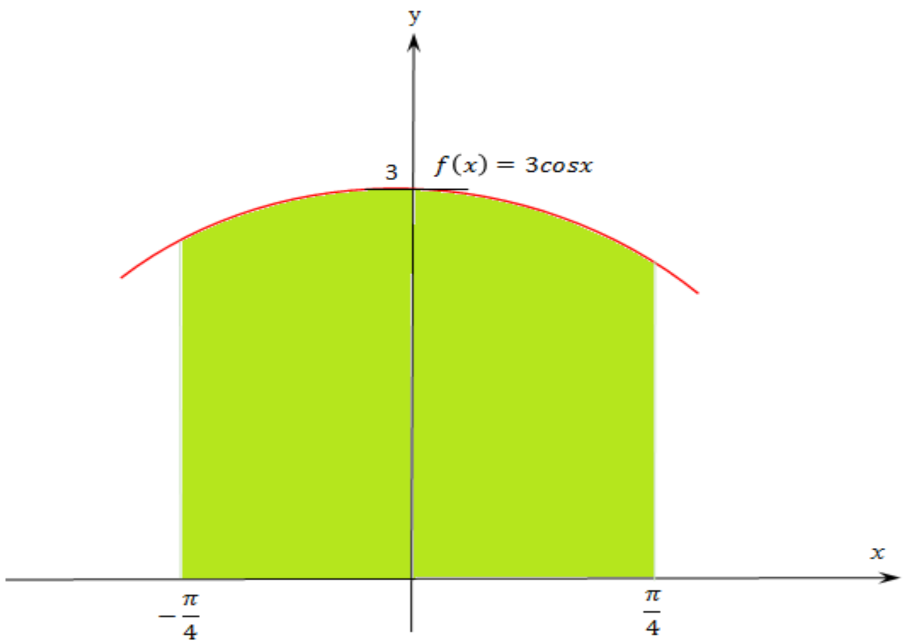
EG02021 Mathematics
Class 7: Integration

5.3 The Fundamental Theorem of Calculus

- **Second Form of the Fundamental Theorem of Calculus**

Exercise 1: Compute the definite integrals-

a) $\int_0^2 (x + x^2)dx$
b) $\int_0^1 e^x dx$
c) $\int_{\pi/2}^{\pi} sinx\, dx$



Exercise 2: Find the area under the graph $f(x) = 3cosx$ over the interval $\left[-{}^{\pi}/_{4}, {}^{\pi}/_{4}\right]$.

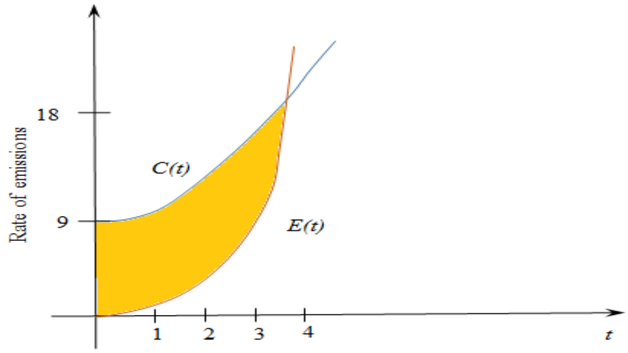
- **The Average Value of a Function**

Exercise 1: Engine Emissions. The emissions of an engine are given by $E(t) = 2t^2$, where $E(t)$ is the engine's rate of emission, in billions of pollution particulates per year, at time *t*, in years. Find the average emissions from t=1 to t=5.

5.4 Properties of Definite Integrals

- **The Area of a Region Bounded by Two Graphs**

Exercise: Emission Control. A clever college student develops an engine that is believed to meet federal standards for emission control. The engine's rate of emission is given by $E(t) = 2t^2$, where *E(t)* is the emissions, in billions of population particulates per year, at time *t*, in years. The emission rate of a conventional engine is given by $C(t) = 9 + t^2$.
The graphs of both curves are shown below.



a) At what point in time will the emission rates be the same?
b) What is the reduction in emissions resulting from using the student's engine between time 0 and when the emission rates are the same?

5.5 Integration Techniques: Substitution

Exercise

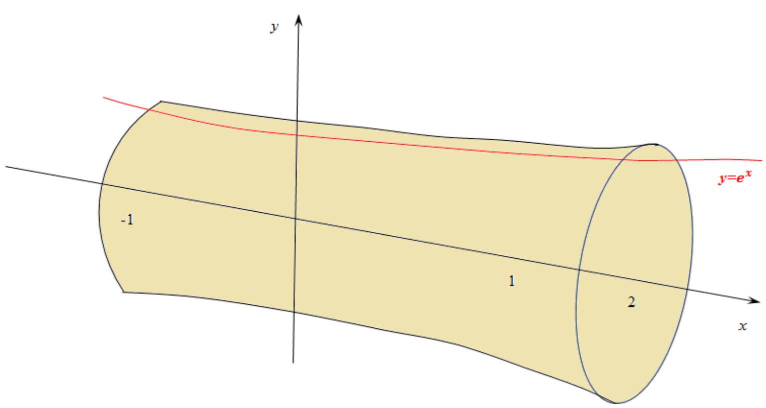
1. Compute $\int 2xe^{x^2}\,dx$.
2. Use substitution and the *dx* notation to integrate $\int 2xe^{x^2}\,dx$.
3. Evaluate $\int \frac{2x\,dx}{1+x^2}$
4. Evaluate $\int \frac{2x\,dx}{(1+x^2)^2}$
5. Evaluate $\int \frac{ln3x}{x}dx$
6. Evaluate $\int x\sqrt{x^2+1\,dx}$
7. Evaluate $\int \frac{1}{\sqrt{x}} sin\sqrt{x}\,dx$

5.6 Integration Techniques: Integration by Parts

Exercise: Evaluate $\int xlnx\,dx$

5.7 Volume

Exercise: Find the volume of the solid of revolution generated by rotating the region under the graph of $y = e^x$ from $x = -1$ to $x = 2$ about the *x*-axis.



5.8 Improper Integrals

Exercise: Determine whether the following integral is convergent or divergent, and calculate its value if it is convergent: $\int_0^\infty 2e^{-2x}\,dx.$

5.9 Trapezoid Rule

The Trapezoid rule approximation for $\int_a^b f(x)dx$ using n trapezoids is

Tn = $\frac{\Delta x}{2}\,[y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n],$

Where

$\Delta x = \frac{b-a}{n}$, $X_k = a + k\Delta x$ and $y_k = f(X_k)$

The weights are the coefficient 1, 2, 2, 2, … , 2, 1 and the *terms* are the weights multiplied by yk. For example, the weight of y_0 is 1 and the first term is 1. y_0=y_0

Exercise:

Estimate the integral $\int_{-1}^{1}\sqrt{1-x^2}$dx using

a) The Trapezoid rule with n = 4
b) The Trapezoid rule with n = 8

6.0 Simpson's Rule

Simpson's approximation for $\int_a^b f(x)dx$ using n subintervals is

Sn = $\frac{\Delta x}{3}\,[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 \ldots + 2y_{n-2} + 4y_{n-1} + y_n],$

Where n is even,

$\Delta x = \frac{b-a}{n}$, $X_k = a + k\Delta x$ and $y_k = f(X_k)$. To use Simpson's rule, n must be even

Exercise:

Estimate the integral $\int_{-1}^{1}\sqrt{1-x^2}dx$ using Simpson's Rule for n = 8