**EG02041 Advanced Mathematics**

**Class 1: Differentiation and Application of Differentiation**

**❑ Review on Differentiation Techniques**

**2.5 Introduction**

**❑ The Power Rule**

$$[Theorem 1, The power rule, \frac{d}{dx}x^{k}=k.x^{k-1}]$$

1. **Exercise: Evaluate** $\frac{d}{dx}x^{-^{2}/\_{3}}$

❑ **The Derivative of a Sum or a Difference**

$$[Theorem 4, The Sum-Difference rule,$$

$$Sum, \frac{d}{dx}\left[f\left(x\right)+g\left(x\right)\right]=\frac{d}{dx} f\left(x\right)+\frac{d}{dx}g(x)$$

$$Difference, \frac{d}{dx}\left[f\left(x\right)-g\left(x\right)\right]=\frac{d}{dx} f\left(x\right)-\frac{d}{dx}g(x)$$

1. **Exercise:** Evaluate $\frac{d}{dx}\left(24x-\sqrt{x}+\frac{2}{x}\right)$

**❑ Application of Differentiation**

**2.6 Instantaneous Rates of Change**

$Definition, Velocity=v\left(t\right)=\lim\_{h\to 0}\frac{s\left(t+h\right)-s(t)}{h}=s^{'}\left(t\right)$

 $Definition, Acceleration=a\left(t\right)=v^{'}\left(t\right)$

1. **Exercise:** *Velocity.* A Cheetah can attain a speed of over 50 mph ≈73 *ft/sec* in 3 sec. Suppose a cheetah runs in such a way that distance *s* (in feet) from the starting point is a function of time *t* (in seconds) as follows:

$s\left(t\right)=12.5t^{2}$ for $0\leq t\leq 3.$

1. Find the average velocity between times *t=*1 and *t=*3.
2. Find the (instantaneous) velocity when *t*=3.

**2.7 The product and Quotient Rules**

**❑ The Product Rule**

 $[Theorem 5, The Product rule, $

$$Suppose that F\left(x\right)=f\left(x\right).g\left(x\right), where f\left(x\right) and g\left(x\right) are differentiable.$$

 $ F^{'}\left(x\right)= \frac{d}{dx}\left[f\left(x\right).g\left(x\right)\right]= f\left(x\right).[\frac{d}{dx}g\left(x\right)+\left[\frac{d}{dx}f\left(x\right)\right].g\left(x\right).]$

1. **Exercise:** Find$\frac{d}{dx}\left[\left(x^{4}-2x^{3}-7\right)\left(3x^{2}-5x\right)\right].$

**❑ The Quotient Rule**

$$[Theorem 6, The Quotient rule, $$

$$if N\left(x\right) and D\left(x\right) are differentiable and Q\left(x\right)=\frac{N\left(x\right)}{D\left(x\right)}, $$

$$then Q^{'\left(x\right)}=\frac{D\left(x\right).N^{'}\left(x\right)-D'(x).N\left(x\right)}{\left[D(x)\right]^{2}} ]$$

1. **Exercise:** Differentiate:$g\left(t\right)=\frac{tsint+t^{3}}{cost}$

**2.8. Chain Rule**

**❑ Extended Power Rule**

$$[Theorem 8, The Extended Power Rule, $$

$$Suppose that g\left(x\right)is a differentiable function of x. Then for any real number k, $$

$$\frac{d}{dx}[g\left(x\right)]^{k}=k\left[g\left(x\right)\right]^{k-1}.\frac{d}{dx}g\left(x\right).$$

1. **Exercise:** Differentiate:$f\left(x\right)=\sqrt[4]{\frac{x+3}{x-1}}$

❑**Chain Rule**

$$[Theorem 9, The chain Rule, $$

$$If f\left(x\right) and g\left(x\right) are differentiable, then th dericative of the composition f.g is given by$$

$$\frac{d}{dx}\left[f.g\left(x\right)\right]=\frac{d}{dx}\left[f\left(g\left(x\right)\right)\right]=f^{'}\left(g\left(x\right)\right).\frac{d}{dx}g\left(x\right).$$

1. **Exercise:** Compute the derivatives.
2. $\frac{d}{dx}tan⁡(3x+2)$

$$b) \frac{d}{dx}[xcsc\left(3x+2\right)]$$

1. **Exercise:** Compute $f^{'}\left(x\right) where f\left(x\right)=tan⁡(sin2x)$

**2.9 Higher-Order Derivatives**

**9. Exercise:** For $y=(x^{2}+10x)^{20},$ find y′ and y″.

**2.9 Higher-Order Derivatives**

**10. Exercise:** *Simple Harmonic Motion.* The vertical position of a weight suspended by a spring is given by

$y\left(t\right)=10cos⁡(3t+1$)

 where *t* is time measured in seconds and *y(t)* is measured in centimetres.

1. Find the velocity function.
2. Find the acceleration function.
3. How are the position and acceleration functions related?

**3.5 Maximum-Minimum Problems**

**11. Exercise:** *Minimizing Surface Area.*A container form is designing an open-top rectangular box, with a square base, that will hold 108 cubic centimetres (cc). What dimensions yield the minimum surface area? What us the minimum surface area?

**Chapter 3: Application of Differentiation**

1. **Exercise:** *Flights of Homing Pigeons*. It is known that homing pigeons tend to avoid flying over water in the daytime, perhaps because the downdrafts of air over water make flying difficult. Suppose a homing pigeon is released on an island at point C, which is **4m** directly out in the water from point B on the shore. Point B is **12m** down shore from the pigeon’s home loft at point A. Assume that a pigeon requires twice the amount of energy per mile to fly over water than to flying over land. At what angle *θ* should the pigeon fly toward the shore in order to minimize the total energy required to get to its home loft?

B

θ

S

A

C

4m

Island

Home Loft

AB=12m

y

*x*

*z*