

EG02041 Advanced Mathematics

Class 9: Probability

10.1 Probability

□ Probability

1. **Exercise: Rolling a Die.** A die is rolled 1000 times. The table in the margin shows the number of times the die came up each possible number. Use this experiment to estimate the probability of each possible outcome.

Face	Frequency
1	166
2	176
3	158
4	178
5	153
6	169
Total	1000

□ Equally Likely Outcomes

2. **Exercise:** A jar contains 7 black balls, 6 yellow balls, 4 green balls, and 3 red balls. The jar is shaken and you remove a ball without looking. What is the probability that (a) the ball is red? (b) that it is white? (c) that it is either black, yellow, green or red?

□ The Addition and Complement Rules

3. **Exercise: Playing Cards.** A card is drawn from a well-shuffled standard deck of 52 cards.
- What is the probability of drawing an ace?
 - What is the probability of drawing a king?
 - What is the probability of not drawing a king?
 - What is the probability of drawing an ace or a king?
4. **Exercise:** A jar contains 7 black balls, 6 yellow balls, 4 green balls, and 3 red balls. The jar is well shaken and a ball is drawn from the jar.
- Find the probability that the ball is yellow.
 - Find the probability that the ball is red.
 - Find the probability that ball is red or yellow.

10.2 Multiplication Tress and Bayes' Rule

□ Genetics

5. **Exercise: Genetics.** Suppose that a pea plant of genotypes FF and ff are crossed. A sperm allele is transmitted from the FF plant, while an ovum allele is transmitted from the ff plant. .
- Find the probability that an offspring is of each of the three genotypes.
 - Find the probability that the offspring will have purple flowers.
 - Find the probability that the offspring will have white flowers.

□ Bayes' Rule

$$P(D|T+) = \frac{P(D+)P(T+|D+)}{P(D+)P(T+|D+) + P(D-)P(T+|D-)}$$

Similarly formulas may be written for $P(D-|T+)$, $P(D+|T-)$ and $P(D-|T-)$.

Bayes' Rule is used to update the probability of an event (like occurrence of a disease) given extra information (like a positive test). It is often easier to formulate Bayes' Rule using a multiplication tree.

6. **Exercise: Diagnosing Coronary Artery Disease.** A man in his 20s with no available family history sees a doctor complaining of chest pain. Because of his age, the doctor estimates the probability that the patient has CAD as 0.05. The patient is then given the echocardiography test of Example 4, and the test returns positive. Find the probability that the patient has CAD given the positive test result.

10.3 The Binomial Distribution

□ Random Variables

10.4 Expected Value and Standard Deviation for Discrete Random Variables

□ Expected Value

7. **Exercise: Rolling a Die.** A die is rolled 1000 times. The table in the margin shows the number of times the die came up each possible number. Use this experiment to find the average value of the 1000 rolls.

Face	Frequency
1	166
2	176
3	158
4	178
5	153
6	169
Total	1000

10.5 Continuous Random Variables

Suppose we throw a dart at a number line in such a way that it always lands in the interval $[1, 3]$. Let X be the number that the dart hits. There are an infinite number of possibilities for X . Note that X is a random variable whose possible values consist of an entire interval of real numbers. Such a random variable is called continuous random variable.

□ Observed Significance Levels

□ Critical Values

□ Constructing Probability Density Functions